9. (a) Verify Cayley-Hamilton theorem for the matrix :

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 4 \\ 2 & 0 & 3 \end{bmatrix}$$

(b) Show that diagonal elements of a Hermitian matrix are all real.

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MATHEMATICAL FOUNDATION-I

BCA-105

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting at least *one* question from each Section and all questions carry equal marks.

1. (a) Find one complement of each element of the lattice D_{35}.

(b) Find the differential equation of $y = Ae^{2x} + Be^{-2x}$. where A and B are constant.

(c) Prove that the proposition $p \vee \sim(p \wedge q)$ is a tautology.

(d) Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

(e) Find $\frac{dy}{dx}$ for $e^x + e^y = e^{x+y}$.

Section I

2. (a) Prove that :

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$

(b) Prove that :

$$^{n}P_{r} = {}^{n-1}P_{r} + r.\, {}^{n-1}P_{r-1}$$

3. (a) Limit of a function at a point if exists is unique.

(b) Differentiate $\tan^{-1}\frac{2x}{1-x^2}$ w.r.t. $\sin^{-1}\frac{2x}{1+x^2}$.

Section II

4. (a) Find the differential equations of all parabolas whose axes are parallel to y-axis.

(b) Solve the diffrential equation :

$$\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$$

5. (a) Solve :

$$\left(x^2+y^2\right)dx - 2xydy = 0$$

(b) Solve the differential equation :

$$x^2\frac{d^2y}{dx^2} - 2y = x^2 + \frac{1}{x}$$

Section III

6. (a) Construct the truth table for :

$$(p \Rightarrow q) \vee \sim(p \Leftrightarrow \sim q)$$

(b) Using mathematical induction to prove that :

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots\ldots+\frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

7. (a) Define an abelian group. If each element of a group is its own inverse, then show that the group is abelian.

(b) Prove that intersection of two subrings is a ring.

Section IV

8. (a) Without using the concept of inverse of a matrix, find the matrix $\begin{bmatrix} x & y \\ z & u \end{bmatrix}$ such that :

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

(b) If $A = \begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$ then find its rank by using elementary operations.